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Some remarks about a paper of Mr. Makar

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I partly agree with the objections of Mr Makar against the paper [3] of Mr Mikhail.

I do not see that the definition of C, and similarly that of b and B are ambiguous; I agree that the notation is somewhat misleading.

Mr Mikhail considers sequences of integers $\{d(n)\}$,such that $0 < d(n) < Dn$ for all n and $\lim_{n\to\infty} \frac{d(n)}{n} = 1$. For such a sequence he defines

$$C\{k, \{d(n)\}\} = \max_i |\pi_{ni}| \, |p_{i,d(k)}| ,$$

$$C\{d(n)\} = \limsup_{k\to\infty} \left[C\{k, \{d(n)\}\}\right]^{\frac{1}{k}} .$$

The last number depends on the choice of the sequence as a whole, but not on a particular index. We now can take the least upper bound of these numbers for all sequences $\{d(n)\}$ as defined above:

$$C = \sup_{\{d(n)\}} C\{d(n)\} .$$

I agree with Mr Makar that the sufficiency part of the proof of theorem 5(b) is not clear. In fact it seems that Mr Mikhail assumes that, given a sequence $\{a_n\}$ of positive integers, one can take three sequences $\{s(n)\}$, $\{d(n)\}$ and $\{g(n)\}$ of positive integers, such that $\lim_{n\to\infty} \frac{s(n)}{n} < 1$, $\lim_{n\to\infty} \frac{d(n)}{n} = 1$ and $\lim_{n\to\infty} \frac{g(n)}{n} > 1$, and such that for every n either $a_n = s(n)$, or $a_n = d(n)$, or $a_n = g(n)$. This statement is not true in general. So the proof contains a gap; I do not see at once how this gap can be filled.

In his paper Mr Makar only refers to the ambiguity of the constants C etc., with which I do not agree. His objections against the proof of theorem 5, with which I agree, are only contained in his note for the referee. So in my opinion his statement on page 2 should be changed, e.g. as follows: The of these numbers C and b in his proof is not completely clea to me.

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